

THE PRODUCTION OF LIGHT GOLDSTONE PARTICLES ON PHOTON COLLIDERS

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Abstract

It is shown that by realizing the project of intensive γ beams with large energy (project PLC) an essential flux of light Goldstone particles (axions, arions, familons, majorons) will be generated. The light higgs can be observed via interaction with matter. The probability of light higgs - electron production by absorption of several laser photons simultaneously is calculated.

1 Introduction

The most important peculiarity of electroweak interaction theory is its renormalizability which is provided by Higgs boson introduction. Their search is considered to be one of the most important problems of elementary particle physics for the nearest future. In fact, there is no special reason to consider that the Higgs sector contains the only one boson. There are some theoretical considerations pointing out the desirability of Higgs sector expanding [1]. In this case in different variants of the theory together with the heavy particles there appear the light pseudogoldstone bosons. In the paper we propose the investigation of new possibilities to study nonstandard Higgs particles such as axion, arion, majoron, familon on the basis of colliding photon-electron bunches.

Nowaday the experiments on the colliding beams have become the source of fundamental information about the matter. On the basis of the future linear colliders (Next Linear Collider) one can make the study of reaction region wider. Lasers available nowadays permit to get the intensive dense photon beams at Compton scattering on the electronic beams [2, 3, 4, 5, 6, 7, 8]. The obtained γe and $\gamma\gamma$ bunches have the same energies and luminosities as the basic electron bunches. For obtaining the intensive photon beams one can use the solid state laser and also the free electron laser. It allows one to study experimentally new problems inaccessible for investigation by other methods.

First, the conversion region is the region of extremely intensive electromagnetic field (a focused laser bunch).

Second, the conversion region can be treated as the $\gamma_0 e$ - collider (γ_0 is the laser photon).

Though the energy of the $\gamma_0 e$ system (in the center-mass system) is not too large, the luminosity of this collider is very large. Let E_e be the energy of electron in the beam, ω_0 is the energy of laser photon. Then

$$W_{\gamma_0 e} = m_e \sqrt{1+x}, \quad x = 4E_e \omega_0 / m_e^2 = 15.3 \left[\frac{E_e}{TeV} \right] \left[\frac{\omega_0}{eV} \right].$$

Note via k - the conversion coefficient $e \rightarrow \gamma$, σ - the cross section of the Compton scattering, N_e - the number of electrons in a bunch, f - the repetition rate, S - the transverse cross section of the electron bunch in the conversion region, N_{γ_0} - the number of laser photon going through the electron bunch. Then the luminosity is

$$L_{\gamma_0 e} = f \frac{N_e N_{\gamma_0}}{S}$$

At the same time, $k \approx N_{\gamma_0} \sigma / S$. Therefore

$$L_{\gamma_0 e} = f \cdot \frac{N_e k}{\sigma} = 10^{38} cm^{-2} s^{-1} \quad (1)$$

Accordingly, the region of laser conversion gives us the possibility to investigate the rare processes of creation of light particles with masses $\leq W_{\gamma_0 e}$ [9, 10, 11, 12].

It is very important to note that these phenomena can be observed with obtaining high energy photons simultaneously. They can also be specially studied on electron beams of existing accelerators such as LEP, SLC and TRISTAN.

The region of laser conversion $e \rightarrow \gamma$ is unique in its physical properties and allows one to study new problems previously inaccessible for investigation - simply as a by-product of obtaining γ beams. In particular it is the creation of light pseudo-Goldstone bosons such as axion, arion, majoron, familon. Due to high luminosity of $n\gamma_0 e$ system the possible mechanism of creation is the processes with simultaneous absorption of few (n) laser photons from laser wave by an electron:

$$n\gamma_0 + e \rightarrow X + e \quad (2)$$

In the paper we study the mechanism of light Goldstone particles (2) production¹. This seems interesting from the following points of view:

- a) the possibility of experimental observation or the set-up of new constraints on the couplings of light particles with leptons and photons;
- b) the influence of the non-linear effects on the probability of creation.

2 The properties of light Goldstone particles and experimental constraints on their couplings

¹ Note the second mechanism of production: $n\gamma_0 + \gamma \rightarrow X$, where γ is the high energy photon, but the number of produced light Goldstone particles is essentially less (contains additional factor m_X^2/m_e^2)

2.1 The "invisible" axion

The light pseudogoldstone boson, named axion, was proposed for the solution of the CP-violation problem in strong interactions [13, 14]. This standard axion is not observed in a number of experiments [15]. The idea of natural CP symmetry is attractive, and the theory of the standard axion was modified to make it interact more weakly with matter and to make it lighter. The axion model with two Higgs doublets is characterized by the scale of breaking of the $U(1)$ -symmetry $f_{pq} \approx 250$ GeV and the mass $m_a \geq 150$ KeV, connected by one parameter: the ratio of the vacuum expectation value (VEV) of Higgs doublets. The introducing of additional Higgs multiplets separates m_a and f_{pq} , and $250 \text{ GeV} \leq f_{pq} \leq 10^{19}$, the mass m_a , as the coupling with matter can become sufficiently small [1]. One of such possibilities is the introduction of the additional scalar field ($SU(2) \times U(1)$ singlet) at arbitrary large VEV. This "invisible" axion is well-known as Dine-Fischler-Srednicki-Zhitnitsky axion (DFSZ) [16, 17]. The interaction Lagrangian of the axion with the electrons and the photons has the form

$$\mathcal{L} = g_{aee} \bar{e} \gamma_5 e a + C_{a\gamma\gamma} \frac{\alpha}{m_e} a F \tilde{F} \quad (3)$$

The couplings g_{aee} and $C_{a\gamma\gamma}$ are given by:

$$g_{aee} = \frac{m_a m_e}{f_\pi m_\pi} \frac{1+z}{N\sqrt{z}} \frac{1}{v^2+1},$$

$$C_{a\gamma\gamma} = \frac{m_a m_e}{8\pi f_\pi m_\pi} \frac{(1+z)}{\sqrt{z}} \left(\frac{8}{3} - \frac{2(4+z)}{3(1+z)} \right),$$

where $f_\pi = 94$ MeV is the pion decay constant, $z = m_u/m_d = 0.568$ is the ratio of the quark masses, N is the number of generations, v is the ratio of the VEV's of two Higgs fields. Respectively, the mass of axion is given by

$$m_a = m_\pi \frac{f_\pi}{f_a} \frac{N\sqrt{z}}{1+z} (v + 1/v).$$

Another possibility of "invisible" axion introduction was proposed in [18, 19], now this axion is well-known as hadronic or Kim-Shifman-Vainstein-Zakharov axion (KSVZ). The KSVZ axion does not interact with leptons at the tree level, because the couplings of interaction to leptons and photons are two orders less compared to DFSZ axion. The archion model can be treated as this class of axion models [20]. This model naturally contains the global symmetry $U(1)$, the spontaneous breaking of which leads to the appearance of Goldstone boson, which has both diagonal and nondiagonal flavor interaction with fermions. But unlike the axion the archion has no interaction with photons and it is like a hadronic axion with strongly suppressed lepton interaction.

The astrophysical constraints on the mass and VEV of "invisible" axion [21]:

$$10^9 \text{ GeV} < f_a < 10^{12} \text{ GeV}, \quad 0.6 \cdot 10^{-5} \text{ eV} < m_a < 0.6 \cdot 10^{-2} \text{ eV}$$

lead to following values of coupling with leptons ($(g_{aee} = m_e/vf_a)$): $g_{aee} = 0.5 \cdot 10^{-12} - 0.5 \cdot 10^{-9}$.

The acceleration data give us more less coupling constraints. In PDG [22] for the Lagrangian $\mathcal{L} = G_{aee} \partial_\mu a \bar{e} \gamma_\mu \gamma_5 e$ the corresponding constraint is $G_{aee} < 2.7 \cdot 10^{-5} \text{ GeV}^{-1}$, that gives for coupling of Lagrangian (3) ($g_{aee} = 2m_e G_{aee}$) $g_{aee} < 3 \cdot 10^{-8}$.

2.2 The Arion

The arion is a neutral, strictly massless, stable pseudoscalar boson with even charge parity, having interactions with fermions [1]. The interaction of an arion with lepton has a form:

$$\mathcal{L} = v_1 \frac{m_e}{v} (i\bar{e}\gamma_5 e)\alpha, \quad v = (G_F\sqrt{2})^{-1/2} = 246 \text{ GeV}.$$

Here α is the arion field, v_1 is the dimensional parameter, equal to the ratio of different VEVs, which theoretical value is of the order 1. However there exist strong restrictions on this value from astrophysical considerations. Weakly interacting with a matter the arions can be emitted from stars. The arion emission leads to fast loss of energy from the stars. The demand of the condition by which the arion luminosity of Sun should not exceed the photon luminosity leads to $v_1 < 10^{-3}$. A more strong constraint appears from the evolution of a red giants: $v_1 < 10^{-6}$. Thus, the coupling with leptons $g_{aee} < 2 \cdot 10^{-9} - 2 \cdot 10^{-6}$.

2.3 The joron

A spontaneously broken global symmetry of lepton number will lead to massive Majorana neutrinos and a Nambu-Goldstone boson the Majoron. This Goldstone can be accomplished by extending the standard model with an additional $SU(2)$ -triplet Higgs multiplet. In Gelmini-Roncadelli model [23] the triplet of Higgs fields with small neutral component VEV provide the small majoron mass of a neutrinos. The Majoron almost consist from neutral triplet component fields, connected only with a neutrino and consist of small admixture of doublet field. Because the majoron weakly interact with a leptons. The Lagrangian of interaction with an electrons has a form

$$\mathcal{L} = 2\sqrt{2}G_F v m_e \bar{e} i \gamma_5 e \Phi_M,$$

where Φ_M is the majoron field, v is VEV. The anomalous coupling of the Majoron to photons vanishes.

The astrophysical constraint from the consideration of Majoron emission rates from the neutron-star core is $v < 2 \text{ KeV}$.²

Thus, we have the following constraint on the coupling with an electron: $g_{\Phi ee} < 3.4 \cdot 10^{-14}$.

2.4 The Familon

The familon is the Goldstone boson associated with the spontaneous breaking of a global family symmetry (horizontal symmetry) [24]. The horizontal symmetry is the spontaneous breaking symmetry between the generation of a quarks and a leptons. Since the breaking of the horizontal group must take place within small distance the familon, similar the invisible axion, interact weakly with the matter and has small mass.

Phenomenologically the effective interaction of a familon with a leptons at low energy can be written in the form:

$$\mathcal{L} = \frac{1}{F} (2m_e \bar{e} \gamma_5 e) \Phi_F.$$

²In another majoron models the constraint on the coupling with the leptons more hard $g_{\Phi_M ee} = 2\sqrt{2}G_F m_e v < 1.7 \times 10^{-18}$.

The astrophysical constraint on the coupling: $F > 7 \cdot 10^9$ GeV, its correspond $g_{Fee} = 2m_e/F < 1.4 \cdot 10^{-13}$. The investigation of decay $K^+ \rightarrow \pi^+ \Phi_F$ gives us the following constraint [22] $B(K^+ \rightarrow \pi^+ \Phi_F) < 1.7 \cdot 10^{-9}$ or $F > 1.3 \cdot 10^{11}$ GeV.

3 The calculations of probability and the number of events.

The interaction Lagrangian of the light Goldstone particles with the electrons in general form can be written as

$$\mathcal{L} = g_{Xee}(\bar{e}\gamma_5 e)\Phi_X.$$

We can consider the field of laser wave with the circularly polarization. The vector potential of a electromagnetic wave has the form $A_\mu = a_{1\mu}\cos(\varphi) + a_{2\mu}\sin(\varphi)$, $\varphi = kx$, where k_μ is the momentum of laser photon, ($ka_1 = ka_2 = a_1a_2 = 0$, $a_1^2 = a_2^2 = a^2$). The matrix element of the light Goldstone by electron in external field can be written in the form

$$M_{fi} = ig_{Xee} \int d^4x \bar{\Psi}_e(x) \gamma_5 \Psi_e \phi_X(x), \quad (4)$$

where $\Psi_e(x)$ ($\bar{\Psi}_e(x)$) are the exact solution of the Dirac equation for an electron in the field of a circularly polarized wave:

$$\Psi_e(x) = (1 + \frac{ek\hat{A}}{2kp})u_p \exp(ie\frac{a_1p}{kp}\sin(\varphi) - ie\frac{a_2p}{kp}\cos(\varphi) + iqx), \quad \phi_X = \frac{e^{-ip_Xx}}{\sqrt{2\epsilon_X}}.$$

Here ϵ_X, p_X is the energy and the momentum of a Goldstone, p and q are the momentum and "quasimomentum" of an electron ³:

$$q_\mu = p_\mu + \xi^2 \frac{m_e^2}{2kp} k_\mu, \quad \xi^2 = \frac{e^2 a^2}{m_e^2}.$$

Using the standard techniques (see [25, 26]), we calculated the probability of Goldstone production by nonpolarized electron

$$dW = \frac{g_{Xee}^2 m_e}{64\pi} \sum W_n \frac{du}{(1+u)^2} \quad (5)$$

$$W_n = \frac{1}{\sqrt{nx+1+\xi^2}} \left(-2\eta^2 J_n^2(z) + \xi^2 \frac{u^2}{1+u} [J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)] \right),$$

where $J_n(z)$ is the Bessel function of n -th order, $u = (kp_a)/(kp')$, p' is the momentum of scattering electron,

$$\eta = \frac{m_X}{m_e}, \quad z = \frac{2\xi}{x} \sqrt{nx+1+\xi^2} \sqrt{\frac{(u_+ - u)(u - u_-)}{(1+u_+)(1+u_-)}}, \quad u_- < u < u_+,$$

³The introduction of quasimomentum of an electron take into account the effective "heavier" of electron in the electromagnetic wave field $m_*^2 = m_e^2(1 + \xi^2)$. It is important for the determination of production threshold.

Figure 1: The dependence $f(x, \xi)$ from x at some values ξ and $\eta = 0$

$$u_{\pm} = \frac{nx + \eta^2 \pm \sqrt{(\eta^2 - nx)^2 - 4\eta^2(1 + \xi^2)}}{2 + nx + 2\xi^2 - \eta^2 \mp \sqrt{(\eta^2 - nx)^2 - 4\eta^2(1 + \xi^2)}}.$$

In formula (5) the term with concrete n corresponds to the Goldstone production by an electron via absorption from electromagnetic wave n laser photons simultaneously (2), n_{th} is the minimal number of photons for the Goldstone production with the mass m_X

$$n_{th} = \frac{1}{x}(\eta^2 + 2\eta\sqrt{1 + \xi^2}) \quad (6)$$

Integrating the expression (5) we get the total production probability of a light Goldstone:

$$\frac{g_{Xee}^2 m_e}{64\pi} f(x, \xi, \eta).$$

In Figure 1 the dependence of function $f(x, \xi, \eta)$ shown at some values ξ . At $\xi^2 \ll 1$, expanding the Bessel function in series, we get from (5)

$$dW_n = \frac{g_{Xee}^2 m_e (\xi/x)^{2n}}{64\pi n! \sqrt{nx + 1 + \xi^2}} D \frac{du}{(1 + u)^2} \quad (7)$$

$$D = -2\eta^2((1 + \xi^2)(u_+ - u)(u - u_-))^n + \frac{x^2 n^2 u^2}{1 + u} ((1 + \xi^2)(u_+ - u)(u - u_-))^{n-1}$$

For $n = 1$, integrating (7) by u , we get the cross-section of creation, calculated earlier in [9]:

$$\sigma = \frac{1}{2} \frac{\alpha g_{Xee}^2}{m_e^2} \frac{1}{x} \left\{ \left(1 - 2\frac{\eta^2}{x} + \frac{2\eta^2(\eta^2 - 2)}{x^2} \right) \ln \left(\frac{4(x + 1)}{(2 + x - \eta^2 + \sqrt{(x - \eta^2)^2 - 4\eta^2})} \right) \right.$$

$$+\sqrt{(x-\eta^2)^2-4\eta^2}\left(1-\frac{7}{2}\eta^2+\frac{3}{2}x-8\frac{\eta^2}{x}-7\frac{\eta^2}{x^2}\right)\Bigg\}$$

At $x = 5$ the cross-section is:

$$\sigma \approx g_{aee}^2 \cdot 5.4 \cdot 10^{-24} \text{ cm}^2$$

For numerical calculation, the physical parameters characterizing the region of laser conversion are chosen in accordance with the projects PLC [5, 7, 8]. In the conversion scheme, it has been proposed to use a solid-state laser with laser photon energy $\omega_0 = 1.17$ eV. The length of conversion region characterized by a high density of laser photons is $l \sim 0.15$, which value is close to the length of the electron bunch. The invariant mass of the $\gamma_0 e$ -system for the energy of electron $E_e = 250$ GeV is comparable with the electron mass $W_{\gamma_0 e} \approx 1.21$ eV ($x = 4.5$). It is convenient to express ξ^2 in terms of the energy A , the duration τ and radius a_γ of the laser flash in the interaction point

$$\xi^2 = \frac{A}{A_*}, \quad \text{where} \quad A_* = \frac{\tau c}{4} \cdot \left(\frac{m_e \omega_0 a_\gamma c}{e \hbar} \right)^2.$$

For the values $\pi a_\gamma^2 \approx 10^{-5} \text{ cm}^2$, $A_* = 100$ J and at the energy of laser flash $A = 25$ J $\xi^2 = 0.25$. Really the value ξ^2 can be reach 0.6. The number of produced Goldstones is

$$N_X = \frac{N_e \tau}{2} \sum_{n > n_{th}} \int_{u_-}^{u_+} dW_n \quad (8)$$

The Goldstone energies are distributed in the interval

$$\frac{\eta^2}{(x + \eta^2)R} E_e < \epsilon_X < E_e \frac{x + \eta^2}{x + 1} R, \text{ where } R = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4\eta^2(x + 1)}{(x + \eta^2)^2}} \right) \quad (9)$$

For $E_e = 250$ GeV at $m_X = 10$ KeV they correspond to the interval

$$20 \text{ MeV} < \epsilon_X < 208 \text{ GeV}$$

. Since the effective mass of the $\gamma_0 e$ -system not large the characteristic emission angles of Goldstones relative to the direction of the electron bunch are $\leq m_e/E_e \approx 10^{-5}$. Therefore the angular spread of Goldstones is defined by the angular spread of electrons in the beam ($\approx 10^{-4}$).

The numerical calculation of the number of Goldstone particles and the couplings are shown in the table.

Table

	g_{Xee}	Cross-section $\sigma(\text{cm}^2)$	The number of events per year
Standard axion	$2 \cdot 10^{-6}$	$2.2 \cdot 10^{-35}$	$7 \cdot 10^{10}$
"Invisible" axion	$3 \cdot 10^{-8}$	$4.9 \cdot 10^{-39}$	$1.5 \cdot 10^7$
Arion	$2 \cdot 10^{-6}$	$2 \cdot 10^{-35}$	$7 \cdot 10^{10}$
Familon	$1.4 \cdot 10^{-13}$	$1 \cdot 10^{-49}$	$3 \cdot 10^{-4}$
Majoron	$3.4 \cdot 10^{-14}$	$5 \cdot 10^{-51}$	$1.5 \cdot 10^{-5}$

The numerical estimations show that by obtaining intensity high energy photons the large number of light Higgs particles will generate.

4 The registration

To registrarate the new particles it is necessary to build the special detectors. For detection it is proposed to use production of hadrons in collisions of produced Goldstones with nuclei of the pin-type lead rod with radius ≈ 2 cm and length ≈ 100 m , placed after a shield to get rid of the background:

$$X + Pb \rightarrow h \quad (10)$$

This reaction will be observed as the production of hadron jets with total energy $\sim \epsilon_X$ and transverse momentum $p_\perp \sim 300$ MeV/c.⁴ Let us evaluate the number of events as an example of standard axion. The cross-section of reaction (10) is $\sim N$ times as large as the cross-section of the axion–nucleon interaction, σ_{an} , where N is the number of nucleons in a nucleus. The cross-section $\sigma_{an} \approx f_{a\pi}\sigma_{\pi n}(v/f_{pq})^2$, where $f_{a\pi}$ the amplitude of the axion–pion transition for the standard axion, $f_{a\pi} = 2 \cdot 10^{-4}$ [13]. Therefore

$$\sigma_{a+Pb \rightarrow h} \approx N f_{a\pi}^2 (v/f_{pq})^2 \sigma_{\pi n} \approx 5 \cdot 10^{-34} \text{ cm}^2.$$

It corresponds to the fact that on the path of lead of 100 m will be observed one event (2), (10) per hour. The increase of additional $U(1)$ symmetry scale on the order give the decrease of the events number in lead of 4 orders, because we really think it is possible to reach $f_{pq} \sim 10$ TeV in this experiment. The background for the reaction (2), (10) will be produce via the high energy photon interaction with the matter of detector. These neutrinos are obtained from hadrons decay between collisions. The energy of creating of such away a neutrinos will be less than ϵ_X and their angle spread will be wide enough, because the background is weak in the pin-type rod.

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⁴The cross-section of lepton pairs production in Bete-Haitler reaction $X + Pb \rightarrow e^+e^- + \dots$, $X + Pb \rightarrow \mu^+\mu^- + \dots$ approximately an order less then cross-section (10)

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